

Higher Spin Holography and AdS String Sigma-Model

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Abstract

We analyze cubic spin 3 interaction in AdS space using the higher spin extension of string-theoretic sigma-model constructed in our previous work, which low energy limit is described by AdS vacuum solution. We find that, in the leading order of the cosmological constant, the spin 3 correlator on the AdS_4 string theory side reproduces the structure of 3-point function of composite operators, quadratic in free fields, in the dual $d = 3$ vector model. The cancellation of holography violating terms in $d = 3$ is related to the value of the Liouville background charge in $d = 4$.

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1. Introduction

It is common to think of *AdS/CFT* holography as of duality between semiclassical limit of supergravity with negative cosmological constant and conformal field theory (CFT) living on the boundary of its vacuum solution (AdS space). This, however, is the low energy approximation; in the stronger sense the *AdS/CFT* conjecture means that the correlation functions of physical vertex operators computed in closed string theory in anti-de Sitter background must reproduce the correlators of the corresponding conformally invariant observables on the CFT side [1], [2], [3]. Regardless of space-time dimension, higher spin fields in AdS space (with various symmetries) inevitably have to play critical role in AdS/CFT holography since the overwhelming number of operators on the CFT (gauge theory side), for example, those of the type

$$\sim Tr(\phi_I \nabla_{m_1} \dots \nabla_{m_s} \phi^I) \quad (1)$$

simply have no choice but to match the higher spin objects propagating in AdS space-time (and possibly polarized along the direction of the boundary). In particular, it has been conjectured [4], [5] that, in case of *AdS₄/CFT₃*, the symmetric fields of spin s in *AdS₄* described by Vasiliev's unfolding formalism (e.g. see [6], [7], [8], [9], [10], [11], [12] are dual to the symmetrized objects of the type (1) at the conformal points of the $O(N)$ vector model in $d = 3$ for even values of s and the $U(N)$ model for odd spins. This conjecture has been checked explicitly in important papers [13], [14], [15] and later analyzed in a number of insightful works, e.g. [16], [17], [18], [19], [20] whose results suggest the importance of the free field theory limit in the $O(N)/HS$ duality, despite the fact that Maldacena-Zhiboedov theorem can be circumvented under certain assumptions [18]. Apart from the low-energy limit, the dynamics of these higher spin fields is described by physical vertex operators in open or closed string theories in anti-de Sitter space and their worldsheet correlation functions. In particular, the *AdS/CFT* duality conjecture strongly suggests the existence of infinite tower of *massless* higher spin states in the string spectrum in *AdS* space-time. In practice, however, little is known about AdS string theory dynamics beyond semiclassical limit, since straightforward quantization of string theory in AdS space-time is not known (e.g. see [21]). Another important point is that, in the standard description, the string excitations correspond to the space-time fields in the metric [22], [23], [24], [25], [26], [27], [28], [29] rather than unfolded formulation, while it is the unfolded formalism which is the most natural and efficient frame-work to approach the problem of the higher

spin extension of the *AdS/CFT* duality [4], [5], [13], [14], [15], [18]. In one of the recent works [30] we constructed the string-theoretic sigma model based on hidden space-time symmetry generators in RNS formalism, realizing AdS_d isometry group. The model is initially defined in the flat background, however, when perturbed by the vertex operators based on the hidden AdS isometry generators, it flows to the new fixed $2d$ conformal point, corresponding to AdS geometry in space-time. This can be shown by analyzing the conformal beta-function of the sigma-model, resulting in the low-energy effective equations of motions, describing (in the leading order) the AdS_d vacuum solutions of gravity with negative cosmological constant in the Mac Dowell-Mansouri-Stelle-West description [31], [32], [33]. Remarkably, the closed string vertex operators, constructed in [30] describe the gravitational excitations around the AdS vacuum in the frame, rather than metric formalism - i.e. in terms of the vielbein and spin connection gauge fields. In this paper we extend our analysis of this sigma-model to include the excitations corresponding to massless higher spin fields in the frame-like Vasiliev's approach. The string-theoretic vertex operators for the frame-like higher spin fields have been constructed in our earlier work [34] where we performed their BRST analysis and analyzed their correlators in flat space, showing them to lead to Berends - Burgers - Van Dam type [35] cubic spin 3 interactions [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48] in flat space. In the current paper we extend this analysis using the sigma-model approach [30] in order to study the AdS deformations of these cubic interactions and their relevance to higher spin holography problem [18], [13], [14], [15], [16], [17], [19], [20], [49], [50], [51], [52], in the context of [4]. We find that, in the leading nontrivial order in ρ^{-1} , the cubic spin 3 interaction reproduces the correlators of the operators of type (1) in the free field limit of the $U(N)$ model in $d = 3$. In particular, in this limit the cubic spin 3 interaction is dominated by the 9-derivative terms, while the lower derivative terms (posing a potential threat to the holography) are absent as their cancellation is ensured by the ghost number selection rules for the vertex operators and by the value of the Liouville background charge ($q = \sqrt{\frac{5}{2}}$) in $d = 4$. The terms with the lower number of derivatives, however, are generally present in the sigma-model for $d \neq 4$. In addition, in the $d = 4$ case these terms may still appear in the higher order corrections in α' (corresponding to $\frac{1}{N}$ corrections in the dual theory). Next, it is the momentum behaviour and the pole structure of the string-theoretic spin 3 amplitude in the sigma-model that corresponds to the field-theoretic pole structure of the 3-point amplitude of the operators (1) of the dual theory in the momentum space. In the following sections, we shall use the AdS string sigma-model to perform the explicit

computation of the 3-point correlators using the vertex operators for spin 3 fields in the frame-like formalism (in the leading nontrivial orders in Λ and α') and discuss physical implications of our results.

2. Sigma-Model for AdS Strings and Vertex Operators for Frame-like Higher Spin Fields: a Brief Review

The sigma-model for AdS strings constructed in [30] is based on hidden space-time symmetry generators in RNS superstring theory. Namely, consider the RNS superstring theory in flat space with the action given by:

$$\begin{aligned}
S_{RNS} &= S_{matter} + S_{bc} + S_{\beta\gamma} + S_{Liouville} \\
S_{matter} &= -\frac{1}{4\pi} \int d^2z (\partial X_m \bar{\partial} X^m + \psi_m \bar{\partial} \psi^m + \bar{\psi}_m \partial \bar{\psi}^m) \\
S_{bc} &= \frac{1}{2\pi} \int d^2z (b \bar{\partial} c + \bar{b} \partial \bar{c}) \\
S_{\beta\gamma} &= \frac{1}{2\pi} \int d^2z (\beta \bar{\partial} \gamma + \bar{\beta} \partial \bar{\gamma}) \\
S_{Liouville} &= -\frac{1}{4\pi} \int d^2z (\partial \varphi \bar{\partial} \varphi + \bar{\partial} \lambda \lambda + \partial \bar{\lambda} \bar{\lambda} + \mu_0 e^{B\varphi} (\lambda \bar{\lambda} + F))
\end{aligned} \tag{2}$$

where φ, λ, F are components of super Liouville field and the Liouville background charge is

$$q = B + B^{-1} = \sqrt{\frac{9-d}{2}} \tag{3}$$

The ghost fields b, c, β, γ are bosonized according to

$$\begin{aligned}
b &= e^{-\sigma}, c = e^{\sigma} \\
\gamma &= e^{\phi-\chi} \equiv e^{\phi} \eta \\
\beta &= e^{\chi-\phi} \partial \chi \equiv \partial \xi e^{-\phi}
\end{aligned} \tag{4}$$

and the BRST charge is

$$\begin{aligned}
Q &= Q_1 + Q_2 + Q_3 \\
Q_1 &= \oint \frac{dz}{2i\pi} (cT - bc\partial c) \\
Q_2 &= -\frac{1}{2} \oint \frac{dz}{2i\pi} (\gamma \psi_m \partial X^m - q \partial \lambda) \\
Q_3 &= -\frac{1}{4} \oint \frac{dz}{2i\pi} b \gamma^2
\end{aligned} \tag{5}$$

Then, in the limit $\mu_0 \rightarrow 0$ the action (2) is symmetric under the global space-time transformations generated by

$$\begin{aligned} T^m &= \frac{1}{\rho} K \circ \oint dz e^\phi (\lambda \partial^2 X_m - 2 \partial \lambda \partial X^m) \\ T^{mn} &= K \circ \oint dz \psi^m \psi^n \end{aligned} \quad (6)$$

where the homotopy transform of an operator V $K \circ V$ is defined according to

$$\begin{aligned} K \circ V &= T + \frac{(-1)^N}{N!} \oint \frac{dz}{2i\pi} (z-w)^N : K \partial^N W : (z) \\ &+ \frac{1}{N!} \oint \frac{dz}{2i\pi} \partial_z^{N+1} [(z-w)^N K(z)] K \{Q_{brst}, U\} \end{aligned} \quad (7)$$

where w is some arbitrary point on the worldsheet, U and W are the operators defined according to

$$[Q_{brst}, V(z)] = \partial U(z) + W(z), \quad (8)$$

$$K = c e^{2\chi - 2\phi} \quad (9)$$

is the homotopy operator satisfying $\{Q_{brst}, K\} = 1$ and N is the leading order of the operator product

$$K(z_1)W(z_2) \sim (z_1 - z_2)^N Y(z_2) + O((z_1 - z_2)^{N+1}) \quad (10)$$

The operators T^m and T^{mn} then can be shown to satisfy the AdS isometry algebra with the cosmological constant $\Lambda = -\frac{1}{\rho^2}$ [30] :

$$\begin{aligned} [T_{ab}, T_{cd}] &= \eta_{ac} T_{bd} - \eta_{ab} T_{cd} - \eta_{cd} T_{ab} + \eta_{bd} T_{ac} \\ [T_a, T_{bc}] &= \eta_{ab} T_c - \eta_{ac} T_b \\ [T_a, T_b] &= -\frac{1}{\rho^2} T_{ab} \end{aligned} \quad (11)$$

The minus sign in the last commutator is actually highly nontrivial and is related to the subtleties of OPE and picture equivalence relations analyzed in [30]. The AdS isometry algebra (11) also admits another realization in terms of the operators (S^m, L^{mn}) where

$$L^{mn} = K \circ T^{mn} \equiv K \circ \oint \frac{dz}{2i\pi} \psi^m \psi^n \quad (12)$$

is the same full rotation operator (6) (where the $K \circ$ represents the homotopy transformation to ensure the BRST-invariance) while S^m is the homotopy transformation of the operator $\oint \frac{dz}{2i\pi} \lambda \psi^m$, representing the rotation in the Liouville-matter plane:

$$\begin{aligned} S^m &= K \circ \rho^{-1} \oint \frac{dz}{2i\pi} \lambda \psi^m \\ &= \rho^{-1} \oint \frac{dz}{2i\pi} [\lambda \psi^m + 2ce^{X-\phi}(\partial\varphi\psi^m - \partial X^m\lambda - qP_{\phi-\chi}^{(1)}\psi^m) - 4\partial cce^{2X-2\phi}\lambda\psi^m] \\ &= -4\{Q, \rho^{-1} \oint \frac{dz}{2i\pi} ce^{2X-2\phi}\lambda\psi^m\} \end{aligned} \quad (13)$$

The conformal weight n polynomials $P_{a\phi+b\chi+c\sigma}^{(n)}$ (where a, b, c are some constants) are defined according to

$$P_{a\phi+b\chi+c\sigma}^{(n)} = e^{-a\phi(z)-b\chi(z)-c\sigma(z)} \frac{d^n}{dz^n} e^{a\phi(z)+b\chi(z)+c\sigma(z)} \quad (14)$$

(with the product taken in algebraic rather than OPE sense)

Starting from the symmetry generators (6), (13), one can construct the closed string vertex operator describing the dynamics of vielbeins and spin connection gauge fields in space-time [30]

$$G(p) = e_m^a(p)F_a\bar{L}^m + \omega_m^{ab}(p)(F_b^m\bar{L}_a - \frac{1}{2}F_{ab}\bar{L}^m) + c.c. \quad (15)$$

where

$$\begin{aligned} F_m &= -2K_{U_1} \circ \int dz \lambda \psi_m e^{ipX}(z) \\ U_1 &= \lambda \psi_m e^{ipX} + \frac{i}{2} \gamma \lambda ((\vec{p}\vec{\psi})\psi_m - p_m P_{\phi-\chi}^{(1)}) e^{ipX} \end{aligned} \quad (16)$$

or manifestly

$$\begin{aligned} F_m &= -2 \int dz \{ \lambda \psi_m (1 - 4\partial cce^{2X-2\phi}) + \\ &2ce^{X-\phi}(\lambda\partial X_m - \partial\varphi\psi_m + q\psi_m P_{\phi-\chi}^{(1)} - \frac{i}{2}((\vec{p}\vec{\psi})\psi_m - p_m P_{\phi-\chi}^{(1)})) \} e^{ipX}(z) \end{aligned} \quad (17)$$

where the *partial* homotopy transform $T \rightarrow L = K_{\Upsilon} \circ T$ of an operator T based on Υ is defined according to

$$\begin{aligned} L(w) &= K_{\Upsilon} \circ T = T + \frac{(-1)^N}{N!} \oint \frac{dz}{2i\pi} (z-w)^N : K \partial^N \Upsilon : (z) \\ &+ \frac{1}{N!} \oint \frac{dz}{2i\pi} \partial_z^{N+1} [(z-w)^N K(z)] K\{Q_{brst}, U\} \end{aligned} \quad (18)$$

where N is the leading order of the OPE of K and Υ . Particularly, if $[Q, T] = \oint \Upsilon$, the partial homotopy transform obviously coincides with the usual homotopy transform. Next,

$$\begin{aligned} \bar{L}^a = & \int d\bar{z} e^{-3\bar{\phi}} \{ \bar{\lambda} \bar{\partial}^2 X^a - 2\bar{\partial} \bar{\lambda} \bar{\partial} X^a \\ & + ip^a (\frac{1}{2} \bar{\partial}^2 \bar{\lambda} + \frac{1}{q} \bar{\partial} \bar{\varphi} \bar{\partial} \bar{\lambda} - \frac{1}{2} \bar{\lambda} (\bar{\partial} \bar{\varphi})^2 + (1 + 3q^2) \bar{\lambda} (3\bar{\partial} \bar{\psi}_b \bar{\psi}^b - \frac{1}{2q} \bar{\partial}^2 \bar{\varphi})) \} e^{ipX} \end{aligned} \quad (19)$$

at the minimal negative picture -3 representation
and

$$\begin{aligned} \bar{L}^a = & K \circ \int d\bar{z} e^{\bar{\phi}} \{ \bar{\lambda} \bar{\partial}^2 X^a - 2\bar{\partial} \bar{\lambda} \bar{\partial} X^a \\ & + ip^a (\frac{1}{2} \bar{\partial}^2 \bar{\lambda} + \frac{1}{q} \bar{\partial} \bar{\varphi} \bar{\partial} \bar{\lambda} - \frac{1}{2} \bar{\lambda} (\bar{\partial} \bar{\varphi})^2 + (1 + 3q^2) \bar{\lambda} (3\bar{\partial} \bar{\psi}_b \bar{\psi}^b - \frac{1}{2q} \bar{\partial}^2 \bar{\varphi})) \} e^{ipX} \end{aligned} \quad (20)$$

at the minimal positive picture $+1$ representation. (similarly for its holomorphic counterpart L^a) Then,

$$F_{ma} = F_{ma}^{(1)} + F_{ma}^{(2)} + F_{ma}^{(3)} \quad (21)$$

where

$$\begin{aligned} F_{ma}^{(1)} = & -4q K_{U_2} \circ \int dz c e^{\chi - \phi} \lambda \psi_m \psi_a \\ U_2 = & [Q - Q_3, c e^{\chi - \phi} \lambda \psi_m \psi_a e^{ipX}] - \frac{i}{2} c \lambda ((\vec{p} \vec{\psi}) \psi_a \psi_m - p_m \psi_a P_{\phi - \chi}^{(1)}) e^{ipX}(z) \end{aligned} \quad (22)$$

$$F_{ma}^{(2)} = K \circ \int dz \psi_m \psi_a e^{ipX} = -4 \{ Q, \int dz c e^{2\chi - 2\phi} e^{ipX} \psi_m \psi_a(z) \} \quad (23)$$

and

$$F_{ma}^{(3)} = K \circ \int dz e^{\phi} (\psi_{[m} \partial^2 X_{a]} - 2\partial \psi_{[m} \partial X_{a]}) e^{ipX}(z) \quad (24)$$

In the limit of zero momentum the holomorphic and the antiholomorphic components of the operator correspond to AdS isometry generators in different realizations, described above. The BRST invariance imposes the following on-shell constraints on vielbein and connection fields:

$$\begin{aligned} p^{[n} e_m^b(p) - \omega_m^{b[n}(p) &= 0 \\ p_{[n} \omega_m^{ab}(p) &= 0 \\ p^m e_m^b(p) &= 0 \\ p^m \omega_m^{ab}(p) &= 0 \end{aligned} \quad (25)$$

The first two constraints represent the linearized equations $R^{AB} = 0$ (the first one being the zero torsion constraint $T^a = R^{a\hat{d}} = 0$ while the second reproducing vanishing Lorenz curvature $R^{ab} = 0$). The last two constraints represent the gauge fixing conditions related to the diffeomorphism symmetries. The fact that the BRST invariance leads to space-time equations in a certain gauge is not surprising if we recall that similar constraints on a standard vertex operator of a photon also lead to Maxwell's equations in the Lorenz gauge. Provided that the constraints (25) are satisfied, the vertex operator $G(p)$ can be written as a BRST commutator in the large Hilbert space plus terms that are manifestly in the small Hilbert space, according to

$$\begin{aligned}
G(p) &= \{Q, W(p)\} + \frac{1}{q} K \circ \omega_m^{ab} \int dz e^\phi (\psi^{[m} \partial^2 X_{a]} - 2 \partial \psi_{[m} \partial_{a]}^X) e^{ipX}(z) \bar{L}_b + c.c. \\
W(p) &= 8e_m^a(p) \bar{L}_a \int dz c \partial \xi \xi e^{-2\phi} \lambda \psi^m e^{ipX} \\
&\quad + \omega_m^{ab} \bar{L}_b \left[-\frac{4}{q} \int dz c \partial \xi \xi e^{-2\phi} \psi_a \psi^m e^{ipX} \right. \\
&\quad \left. + 4 \int dz (z-w) \partial c c \partial^2 \xi \partial \xi \xi e^{-3\phi} \lambda \psi_a \psi^m e^{ipX} \right]
\end{aligned} \tag{26}$$

This particularly implies that , modulo gauge transformations, the vertex operator $G(p)$ is the element of the *small* Hilbert space.

The linearized gauge symmetry transformations for vielbein and connection gauge fields are given by:

$$\begin{aligned}
\delta e_m^a &= \partial_m \rho^a + \rho_m^a \\
\delta \omega_m^{ab} &= \partial_m \rho^{ab} + \rho^{[a} \delta_m^{b]}
\end{aligned} \tag{27}$$

where we write $\rho^{AB} = (\rho^{ab}, \rho^{a\hat{d}}) = (\rho^{ab}, \rho^a)$ The variation of $G(p)$ under (27) in the momentum space is

$$\delta G(p) = p^m F_m \bar{L}_a \rho^a + p^m F_{ma} \bar{L}_b \rho^{ab} \tag{28}$$

The two terms of the variation (28) are BRST exact in the *small* Hilbert space (and therefore are irrelevant in correlators) since

$$\begin{aligned}
p^m F_m &= \{Q, : \Gamma : (w) [Q, \xi A] \} \\
A &= \int dz e^{\chi-3\phi} \partial \chi ((\vec{p} \vec{\partial} \vec{X}) \lambda - (\vec{p} \vec{\psi}) \partial \varphi + (\vec{p} \vec{\psi}) P_{\phi-(1+q)\chi}^{(1)}) e^{ipX}
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
p^m F_{ma}^{(1)} &= 4q[Q, \Gamma(w) \int dz c e^{-3\phi} \partial \xi \partial^2 \xi \lambda \psi_a (\vec{p} \vec{\psi}) e^{ipX}] \\
p^m F_{ma}^{(2)} &= \{Q, : \Gamma : (w) \int dz \partial \xi e^{-3\phi} ((\vec{p} \vec{\psi}) \partial X_a - (\vec{p} \vec{\partial} \vec{X}) \psi_a) e^{ipX} \} \\
p^m F_{ma}^{(3)} &= \{Q, [K \circ \int dz \lambda \psi_a e^{ipX}, B]\} \\
B &= \int dz \partial \xi e^{-4\phi} [\lambda (\partial \vec{\psi} \partial^2 \vec{X}) - 2 \partial \lambda ((\vec{\psi} \partial^2 \vec{X}) - 2 (\partial \vec{\psi} \partial \vec{X}))]
\end{aligned} \tag{30}$$

Therefore gauge transformations of e and ω shift $G(p)$ by terms not contributing to correlators. The $G(p)$ vertex operator, whose construction is explained above, describes the dynamics of spin $s = 2$ massless field in the closed string spectrum, in terms of vielbein and connection gauge fields. Note that, despite the fact that the unperturbed theory has been originally defined in the flat space, in the *perturbed* theory (which flows to the AdS vacuum) the distinction between the tangent indices a, b, \dots and the manifold indices m, n, \dots is ensured by the corresponding operators of the F_m -type and L^a -type being the elements of different ghost cohomologies and having very different on-shell constraints and gauge symmetries. In the leading order, the vanishing *beta*-function condition for the sigma-model, given by the RNS action perturbed by the $G(p)$ -operator (15) leads to space-time equations for ω and λ , given by [30]

$$R^{ab} = d\omega^{ab} + (\omega \wedge \omega)^{ab} - \frac{1}{\rho^2} e^a \wedge e^b = 0 \tag{31}$$

and

$$de^a + \omega^{ab} \wedge e^b = 0 \tag{32}$$

with the solution given by the *AdS* vacuum [31]. From the 2-dimensional point of view this means that the RNS theory, initially defined in the flat space and perturbed with $G(p)$, flows to new fixed conformal point corresponding to theory in the new space-time background, namely, *AdS*. The next step is to introduce the higher spin excitations. The vertex operators, describing the dynamics of the massless higher spin $s \geq 3$ fields in Vasiliev's frame-like formalism, can be constructed in the *open* string sector of the extended RNS superstring theory. Such a construction has been recently performed in [34]. In the frame-like approach, the spin 3 field is described by the dynamical space-time field $\omega^{2|0} \equiv \omega_m^{ab}$, as well as by two auxiliary fields $\omega^{2|1} \equiv \omega_m^{ab|c}$ and $\omega^{2|2} \equiv \omega_m^{ab|cd}$, related to $\omega^{2|0}$ by generalized zero torsion constraints [34]

The vertex operators for the dynamical $\omega^{2|0}$ -field for the massless spin 3 are given by:

$$V^{(-3)} = H_{abm}(p)ce^{-3\phi}\partial X^a\partial X^b\psi^me^{ipX} \quad (33)$$

at unintegrated minimal negative picture and

$$V^{(+1)} = K \circ H_{abm}(p) \oint dz e^\phi \partial X^a \partial X^b \psi^m e^{ipX} \quad (34)$$

at integrated minimal positive picture +1. These operators are the elements of superconformal ghost cohomology $H_1 \sim H_{-3}$ [34] The vertex operators for the first auxiliary field $\omega^{2|1}$ are given by

$$\begin{aligned} V_-^{2|1}(p) = & 2\omega_m^{ab|c}(p)ce^{-4\phi}(-2\partial\psi^m\psi_c\partial X_{(a}\partial^2X_{b)} \\ & -2\partial\psi^m\partial\psi_c\partial X_a\partial X_b + \psi^m\partial^2\psi_c\partial X_a\partial X_b)e^{ipX} \end{aligned} \quad (35)$$

at negative (unintegrated) representation and

$$\begin{aligned} V_+^{2|1}(p) = & 2\omega_m^{ab|c}(p)K \circ \oint dz e^{2\phi}(-2\partial\psi^m\psi_c\partial X_{(a}\partial^2X_{b)} \\ & -2\partial\psi^m\partial\psi_c\partial X_a\partial X_b + \psi^m\partial^2\psi_c\partial X_a\partial X_b)e^{ipX} \end{aligned} \quad (36)$$

at the positive (integrated) representations. The operators (35), (36) are the elements of $H_2 \sim H_{-4}$; the cohomology constraints for $V_\pm^{2|1}(p)$ lead to generalized zero torsion constraints relating $\omega^{2|1}$ and $\omega^{2|0}$:

$$\omega_m^{ab|c} = 2p^c\omega_m^{ab} - p^a\omega_m^{bc} \quad (37)$$

modulo BRST exact terms in small Hilbert space. The vertex operators for the second auxiliary field $\omega^{2|2}$ are given by

$$\begin{aligned} V_-^{2|2}(p) = & -3\omega_m^{ab|cd}(p)ce^{-5\phi}(\psi^m\partial^2\psi_c\partial^3\psi_d\partial X^a\partial X_b - 2\psi^m\partial\psi_c\partial^3\psi_d\partial X_a\partial^2X_b \\ & + \frac{5}{8}\psi^m\partial\psi_c\partial^2\psi_d\partial X_a\partial^3X_b + \frac{57}{16}\psi^m\partial\psi_c\partial^2\psi_d\partial^2X_a\partial^2X_b)e^{ipX} \end{aligned} \quad (38)$$

at negative (unintegrated) representation and

$$\begin{aligned} V_+^{2|2}(p) = & -3\omega_m^{ab|cd}(p)K \circ \oint dz e^{3\phi}(\psi^m\partial^2\psi_c\partial^3\psi_d\partial X^a\partial X_b - 2\psi^m\partial\psi_c\partial^3\psi_d\partial X_a\partial^2X_b \\ & + \frac{5}{8}\psi^m\partial\psi_c\partial^2\psi_d\partial X_a\partial^3X_b + \frac{57}{16}\psi^m\partial\psi_c\partial^2\psi_d\partial^2X_a\partial^2X_b)e^{ipX} \end{aligned} \quad (39)$$

at positive (integrated) representation. The $V_{\pm}^{2|2}$ operators are the elements of $H_3 \sim H_{-5}$ and the cohomology constraint leads to the second generalized torsion condition relating $\omega^{2|2}(p)$ and $\omega^{2|1}(p)$ up to BRST exact terms:

$$\omega_m^{ab|cd} = 2p^d \omega^{ab|c} - p^a \omega^{bd|c} - p^b \omega^{ad|c} + 2p^c \omega^{ab|d} - p^a \omega^{bc|d} - p^b \omega^{ac|d} \quad (40)$$

. Combining the *AdS* sigma-model construction [30] with expressions for vertex operators describing the higher spin excitations in unfolded formalism, the generating functional for the model describing the higher spin dynamics in AdS space is given by

$$Z(e_m^a, \omega_m^{ab}, \omega^{s-1|t}, \rho) = \int D(X, \psi, \bar{\psi}, ghosts) e^{-S_{RNS} + \int_p \{G(p, \rho) + \sum_s \sum_{t=0}^{s-1} \omega^{s-1|t}(p) V^{s-1|t}(p)\}} \quad (41)$$

where $\{\omega^{s-1|t}\}$ is the set of dynamical and auxiliary fields for the spin $s > 2$ and $V^{s-1|t}$ are the corresponding massless vertex operators in open string theory. In this paper we shall restrict ourselves to the spin 3 case. The correlation functions describing the higher spin interactions in the *AdS* space are then given by

$$\langle V^{s_1-1|t_1}(p_1) \dots V^{s_N-1|t_N}(p_N) \rangle = \frac{\delta^{(n)} Z(e_m^a, \omega_m^{ab}, \omega^{s-1|t}, \rho)}{\delta \omega^{s_1-1|t_1}(p_1) \dots \delta \omega^{s_N-1|t_N}(p_N)} \Big|_{\omega^{s_1-1|t_1}=0, \dots, \omega^{s_N-1|t_N}=0} \quad (42)$$

In the rest of the paper, for the sake of the holographic context, we shall assume that all the operators of the d -dimensional non-critical superstring theory are polarized along the $d-1$ -dimensional subspace and also propagate in this subspace corresponding to the underlying *AdS_d* boundary, unless stated otherwise.

Perturbation expansion in the powers of $\frac{1}{\rho}$ then describes the *AdS* deformations of the higher spin interactions in terms of α' and the cosmological constant Λ in the frame-like formalism. In the next section we shall use the generating functional (41) in order to compute the *AdS* deformations of the 3-vertex for the spin 3 fields in the first nontrivial order in Λ and to analyze their relevance to the CFT correlators in the dual model.

3. Holographic Spin 3 Vertex in AdS background: preliminaries

In the previous work [34] we computed the three-point function of the spin 3 vertex operators (33)-(40) in open string theory in the flat space, showing it to reproduce the Berends-Burgers-Van Dam (BBD) type of interaction vertex in space-time [35] for spin 3 in the frame-like formalism. To compute the *AdS_d* deformations of this vertex, one has to expand the functional (41) in powers of $\frac{1}{\rho}$, that is, $G(p)$. The result significantly depends on the number of space-time dimensions since $G(p)$ expression (15) depends manifestly on

the Liouville background charge. Since $G(p)$ operator for vielbein and spin 2 connection is a closed string excitation and spin 3 fields vertex operators are in the open string spectrum, the leading order contribution to the AdS_d -deformation stems from the amplitude on the disc. Furthermore, it is clear that the contribution linear in the spin 2 connection ω_m^{ab} , which has the order of $\rho^{-1} \sim \sqrt{\Lambda}$ vanishes since the corresponding correlator is linear in the Liouville superpartner λ , i.e. is proportional to the vanishing one-point function of λ . Similarly, all the contributions proportional to odd powers of ρ^{-1} or half-integer powers of Λ vanish as well, since they all are given by the correlators containing odd numbers of λ insertions. For this reason, the first nontrivial leading order contribution to the disc correlator is proportional to the AdS_d vielbein field $e_m^a(p)$ and is of the order of ρ^{-2} . This is the contribution given by the 4-point function on the disc, equivalent to the five-point function on the sphere. The ghost number selection rule therefore requires that the overall left+right ϕ -ghost number carried by the correlator equals to -2 . This selection rule particularly makes it convenient to take two spin 3 operators at the $\omega^{2|0}$ representation and at the negative unintegrated ghost picture -3 representation (33). It is convenient to locate them at the points $z_{1,2} = \pm i$ on the disc. The third spin 3 operator is convenient to take at the $\omega^{2|2}$ -representation (39) and at the minimal positive integrated ghost picture $+3$ representation. The ghost number selection rule then requires that all the AdS_d transvection L_m -type operators in the $G(p)$ insertion must be taken at positive $+1$ picture representation while the transvection F_a -type or the rotation F_{ab} -type operators entering $G(p)$ must be taken at picture 0 (the latter do not of course contribute to the leading order for the reasons described above). The $G(p)$ operator is also integrated over the interior of the disc. Finally, it is convenient to present the manifest form of the transvection type operators entering $G(p)$ and of the integrated spin 3 operator for $\omega^{2|2}$, upon applying all the relevant homotopy/partial homotopy transforms:

$$F_m = -2 \int dz \{ \lambda \psi_m (1 - 4 \partial c c e^{2\chi - 2\phi}) + 2 c e^{\chi - \phi} (\lambda \partial X_m - \partial \varphi \psi_m + q \psi_m P_{\phi - \chi}^{(1)} - \frac{i}{2} ((\vec{p} \vec{\psi}) \psi_m - p_m P_{\phi - \chi}^{(1)})) \} e^{ipX}(z) \quad (43)$$

and

$$L^a(p, u) = \frac{1}{2} \int dz (z - u)^2 \{ (P_{2\phi - 2\chi - \sigma}^{(2)} e^\phi - 24 \partial c c e^{2\chi - \phi}) \times \{ \lambda \partial^2 X^a - 2 \partial \lambda \partial X^a i p^a (\frac{1}{2} \partial^2 \lambda + \frac{1}{q} \partial \varphi \partial \lambda - \frac{1}{2} \lambda (\partial \varphi)^2 + (1 + 3q^2) \lambda (3 \partial \psi_b \psi^b - \frac{1}{2q} \partial^2 \varphi)) + c e^\chi G^{(4)}(\phi, \chi, \psi, \lambda, \varphi, X) \} e^{ipX}(z) \quad (44)$$

where u is an arbitrary point which choice is irrelevant to the correlators since all the u -derivatives of L^a are BRST-exact in the small Hilbert space [53]. For our purposes, it shall be particularly convenient to choose $u = -i$ on the unit disc boundary. Finally

$$\begin{aligned}
V_+^{2|2}(p) = & -3\omega_m^{ab|cd}(p) \oint dz(z-u)^6 \{ (\frac{1}{6!} e^{3\phi} P_{2\phi-2\chi-\sigma}^{(6)} - 28\partial cce^{2\chi+\phi}) \\
& \times (\psi^m \partial^2 \psi_c \partial^3 \psi_d \partial X^a \partial X_b - 2\psi^m \partial \psi_c \partial^3 \psi_d \partial X_a \partial^2 X_b \\
& + \frac{5}{8} \psi^m \partial \psi_c \partial^2 \psi_d \partial X_a \partial^3 X_b + \frac{57}{16} \psi^m \partial \psi_c \partial^2 \psi_d \partial^2 X_a \partial^2 X_b) \\
& + ce^{\chi+2\phi} G^{(12)}(\phi, \chi, \psi, \lambda, \varphi, X) \} e^{ipX}
\end{aligned} \tag{45}$$

where $G^{(4)}(\phi, \chi, \psi, \lambda, X)$ and $G^{(12)}(\phi, \chi, \psi, \lambda, X)$ are certain operators of conformal dimensions 4 and 12 accordingly, depending on derivatives of the matter and Liouville fields X, φ , bosonized ghost fields ϕ and χ and also on λ, ψ and their derivatives. The manifest form of these operators is irrelevant to us since the pieces proportional to $\sim ce^\chi$ in L^a and to $\sim ce^{\chi+2\phi}$ in $V_+^{2|2}(p)$ don't contribute to the overall correlator due to the ghost number selection rules. Similarly, the selection rules exclude the pieces proportional to $\sim \partial cce^{2\chi-\phi}$ and $\sim \partial cce^{\chi+2\phi}$ in the expressions (44), (45) for L^a and $V_+^{2|2}(p)$ accordingly. Finally, the selection rules leave the only relevant term in the expression (17) for F_m proportional to $\sim \oint dz \lambda \psi_m$ with all others not contributing to the leading order correlator for the same reason. This altogether significantly simplifies the computation of the 5-point correlator, making it still cumbersome but not anymore insurmountable.

4. Holographic Spin 3 Vertex in AdS background: the computation

Using the results of the previous section, it is now straightforward to identify the correlator giving the AdS deformation of spin 3 vertex in the leading order:

$$\begin{aligned}
A(p; k_1, k_2, k_3) = & e_m^a(p) \omega_{m_3}^{a_3 b_3 | c_3 d_3}(k_1) \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \\
& \times \{ < \int d^2 z (z-u)^2 \{ e^\phi P_{2\phi-2\chi-\sigma}^{(2)} (\lambda \partial^2 X_a - 2\partial \lambda \partial X_a) \\
& + ip_a (\frac{1}{2} \partial^2 \lambda + 3(1+3q^2) \lambda \partial \psi_b \psi^b) e^{ipX}(z) \bar{\lambda} \bar{\psi}^m e^{ipX}(\bar{z}) + c.c. \} \\
& \int d\tau (\tau - \tau_1)^6 P_{2\phi-2\chi-\sigma}^{(6)} e^{3\phi} (\psi^m \partial^2 \psi_c \partial^3 \psi_d \partial X^a \partial X_b \\
& - 2\psi^m \partial \psi_c \partial^3 \psi_d \partial X_a \partial^2 X_b \\
& + \frac{5}{8} \psi^m \partial \psi_c \partial^2 \psi_d \partial X_a \partial^3 X_b + \frac{57}{16} \psi^m \partial \psi_c \partial^2 \psi_d \partial^2 X_a \partial^2 X_b) e^{ik_1 X}(\tau) \\
& ce^{-3\phi} \psi^{m_1} \partial X_{a_1} \partial X_{b_1} e^{ik_2 X}(\tau_1) ce^{-3\phi} \psi^{m_2} \partial X_{a_2} \partial X_{b_2} e^{ik_2 X}(\tau_2) > \}
\end{aligned} \tag{46}$$

where $\tau_1 = -i, \tau_2 = i$, the τ -integration is over the disc boundary and the z -integral is over the interior of the disc. In order to simplify the computations, the useful strategy is to first perform the conformal transformation from the disc to the upper half-plane using

$$z \rightarrow \frac{i}{2} \frac{z+i}{z-i} \quad (47)$$

. Then the integrand of the correlator (46) can be computed on the half-plane and then integrated in τ (which, upon the conformal transformation, becomes the integral over the real axis). Having done that, we shall conformally map the obtained expression back to the disc, in order to perform the z -integration over the disc's interior. So we start with the first step, that is, computing the integrand of (46) on the half-plane.

The contributions to this correlator are factorized in terms of ghost, $\psi - \lambda$ and X -parts. Let us start with the ghost part, given by

$$A_{gh}(\tau, z, \tau_1, \tau_2) = \langle e^{3\phi} P^{(6)}(\tau) e^{\phi} P^{(2)}_{2\phi-2\chi-\sigma}(z) c e^{-3\phi}(\tau_1) c e^{-3\phi}(\tau_2) \rangle \quad (48)$$

Note that, upon the conformal transformation (47) we have $\tau_1 = 0, \tau_2 \rightarrow \infty$, so as usual, we only need the leading order of this correlator in τ_2 (all others shall result in expressions with negative powers of τ_2 in the overall correlator, vanishing in the limit $\tau_2 \rightarrow \infty$ and corresponding to the pure gauge contributions) This means that we only should consider the contractions of the ghost polynomials $P^{(2)}_{2\phi-2\chi-\sigma}(z)$ and $P^{(6)}_{2\phi-2\chi-\sigma}(\tau)$ between themselves and with the ghost exponents $e^{3\phi}(\tau)$, $e^{\sigma-3\phi}(\tau_1)$ and $e^{\phi}(z)$. First of all, we note that (as it is straightforward to check) the contractions between the ghost polynomials are limited to

$$\begin{aligned} P^{(2)}_{2\phi-2\chi-\sigma}(z) P^{(6)}_{2\phi-2\chi-\sigma}(\tau) = & P^{(2)}_{2\phi-2\chi-\sigma}(z) P^{(6)}_{2\phi-2\chi-\sigma}(\tau) : \\ & - \frac{12}{(z-\tau)^2} P^{(1)}_{2\phi-2\chi-\sigma}(z) P^{(5)}_{2\phi-2\chi-\sigma}(\tau) \end{aligned} \quad (49)$$

Then correlator (48) can be computed using the associate ghost polynomial (AGP) technique, explained in [34]. The table of the relevant associate ghost polynomials for $P^{(6)}_{2\phi-2\chi-\sigma}$ is straightforward to compute and is given by (using the same notations as in [34]):

$$\begin{aligned} P^{0|6}_{2\phi-2\chi-\sigma|\sigma-3\phi} &= 0 \\ P^{1|6}_{2\phi-2\chi-\sigma|\sigma-3\phi} &= 6! P^{(1)}_{2\phi-2\chi-\sigma} \\ 1P^{2|6}_{2\phi-2\chi-\sigma|\sigma-3\phi} &= \frac{5}{2} \times 6! P^{(2)}_{2\phi-2\chi-\sigma} \end{aligned}$$

$$\begin{aligned}
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{3|6} &= \frac{5}{3} \times 6! P_{2\phi-2\chi-\sigma}^{(3)} \\
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{4|6} &= \frac{5}{12} \times 6! P_{2\phi-2\chi-\sigma}^{(4)} \\
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{5|6} &= \frac{211}{24} \times 6! P_{2\phi-2\chi-\sigma}^{(5)} \\
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{6|6} &= P_{2\phi-2\chi-\sigma}^{(6)} \\
P_{2\phi-2\chi-\sigma|\phi}^{0|6} &= 7! \\
P_{2\phi-2\chi-\sigma|\phi}^{1|6} &= -6 \times 6! P_{2\phi-2\chi-\sigma}^{(1)} \\
P_{2\phi-2\chi-\sigma|\phi}^{2|6} &= \frac{5}{2} \times 6! P_{2\phi-2\chi-\sigma}^{(2)} \\
P_{2\phi-2\chi-\sigma|\phi}^{3|6} &= -\frac{2}{3} \times 6! P_{2\phi-2\chi-\sigma}^{(3)} \\
P_{2\phi-2\chi-\sigma|\phi}^{4|6} &= \frac{1}{8} \times 6! P_{2\phi-2\chi-\sigma}^{(4)} \\
P_{2\phi-2\chi-\sigma|\phi}^{5|6} &= \frac{5}{2} \times 6! P_{2\phi-2\chi-\sigma}^{(5)} \\
P_{2\phi-2\chi-\sigma|\phi}^{6|6} &= P_{2\phi-2\chi-\sigma}^{(6)} \\
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{0|5} &= 5! \\
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{1|5} &= 5 \times 5! P_{2\phi-2\chi-\sigma}^{(1)} \\
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{2|5} &= 5 \times 5! P_{2\phi-2\chi-\sigma}^{(2)} \\
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{3|5} &= \frac{5}{3} \times 5! P_{2\phi-2\chi-\sigma}^{(3)} \\
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{4|5} &= \frac{5}{24} \times 5! P_{2\phi-2\chi-\sigma}^{(4)} \\
P_{2\phi-2\chi-\sigma|\sigma-3\phi}^{5|5} &= P_{2\phi-2\chi-\sigma}^{(5)} \\
P_{2\phi-2\chi-\sigma|\phi}^{0|5} &= -6! \\
P_{2\phi-2\chi-\sigma|\phi}^{1|5} &= -\frac{5}{2} \times 6! P_{2\phi-2\chi-\sigma}^{(1)} \\
P_{2\phi-2\chi-\sigma|\phi}^{2|5} &= 3 \times 6! P_{2\phi-2\chi-\sigma}^{(2)} \\
P_{2\phi-2\chi-\sigma|\phi}^{3|5} &= -\frac{19}{12} \times 6! P_{2\phi-2\chi-\sigma}^{(3)} \\
P_{2\phi-2\chi-\sigma|\phi}^{4|5} &= \frac{13}{24} \times 6! P_{2\phi-2\chi-\sigma}^{(4)} \\
P_{2\phi-2\chi-\sigma|\phi}^{5|5} &= P_{2\phi-2\chi-\sigma}^{(5)}
\end{aligned} \tag{50}$$

Using (49) and the table (50) the ghost correlator (48) is straightforward to compute and

is given by:

$$\begin{aligned}
& \frac{1}{2!6!} \lim_{\tau_2 \rightarrow \infty} A_{gh}(\tau, z, \tau_1, \tau_2) = \tau_2^4 (\tau - \tau_1)^9 (z - \tau_1)^3 (\tau - z)^{-3} \\
& \times \left\{ \left[\frac{21}{(\tau - z)^2} + \frac{10}{(\tau_1 - z)^2} + \frac{30}{(\tau - z)(z - \tau_1)} \right] \times \left[\frac{7}{(\tau - z)^6} - \frac{30}{(\tau - z)^5 (\tau - \tau_1)} \right. \right. \\
& + \frac{50}{(\tau - z)^4 (\tau - \tau_1)^2} - \frac{40}{(\tau - z)^3 (\tau - \tau_1)^3} + \frac{15}{(\tau - z)^2 (\tau - \tau_1)^4} - \frac{2}{(\tau - z)(\tau - \tau_1)^5} \left. \right] \quad (51) \\
& - \frac{1}{(\tau - z)^2} \left(\frac{6}{\tau - z} + \frac{5}{z - \tau_1} \right) \times \left[-\frac{6}{(\tau - z)^5} - \frac{75}{(\tau - z)^4 (\tau - \tau_1)} + \frac{360}{(\tau - z)^3 (\tau - \tau_1)^2} \right. \\
& \left. \left. - \frac{570}{(\tau - z)^2 (\tau - \tau_1)^3} + \frac{390}{(\tau - z)(\tau - \tau_1)^4} + \frac{1}{(\tau - \tau_1)^5} \right] \right\}
\end{aligned}$$

This concludes the calculation of the ghost factor of the overall correlator (46). Next, we shall consider the matter factor of the correlator (46). Structurally, the $G(p)$ insertion contributes two different matter pieces: the first resulting from the L_a -factor of $G(p)$ containing the matter factor proportional to $L_a^{(1)} \sim \lambda \partial^2 X_a - 2\partial\lambda\partial X_a + \frac{1}{2}p_a \partial^2 \lambda$ with no ψ -dependence and the second one stemming from the ψ -dependent piece of L_a proportional to $L_a^{(2)} \sim (3 + 9q^2)p_a \lambda \partial\psi_b \psi^b$. Second, the matter part of the $V_+^{2|2}$ consists of two terms of the type $\partial^{(M_1)} X_{a_3} \partial^{(M_2)} X_{b_3} \partial^{(P_1)} \psi^{m_3} \partial^{(P_2)} \psi_{c_3} \partial^{(P_3)} \psi_{d_3}$ with $M_{1,2}$ ranging from 1 to 3, $P_{1,2,3}$ ranging from 0 to 3 and satisfying $M_1 + M_2 + P_1 + P_2 + P_3 = 7$. The structure of the spin 3 interaction is determined by the ψ -contractions between themselves and by the X -contractions between themselves and with the exponents. The total number of the X -fields in the correlator (46) is equal to 7, so generically, their contractions with the exponents may bring from 1 to 7 derivatives in the cubic vertex. Since the $\omega^{2|2}$ field already contains two derivatives, the possible types of X -contractions result in interaction terms with the number of derivatives ranging from 3 to 9. The 9-derivative contribution with maximum number of derivatives (corresponding to the case when all the X -derivatives contract to the exponents) is of particular interest to us since, in the case of AdS_4 , this contribution is related to the holographic correspondence with the $d = 3$ vector model correlator of the type

$$A(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \langle \Phi_I \partial_{m_1} \partial_{a_1} \partial_{b_1} \Phi^I(\vec{x}_1) \Phi_J \partial_{m_2} \partial_{a_2} \partial_{b_2} \Phi^J(\vec{x}_2) \Phi_K \partial_{m_3} \partial_{a_3} \partial_{b_3} \Phi^K(\vec{x}_3) \rangle \quad (52)$$

in the dual vector model; note that

$$\langle \Phi_I \Phi^I(\vec{x}_1) \Phi_J \Phi^J(\vec{x}_2) \Phi_K \Phi^K(\vec{x}_3) \rangle \sim N |\vec{x}_1 - \vec{x}_2|^{-1} |\vec{x}_1 - \vec{x}_3|^{-1} |\vec{x}_2 - \vec{x}_3|^{-1}. \quad (53)$$

So let us concentrate on the 9-derivative case first and on its relevance to the AdS_4/CFT_3 duality. Since we are interested in relating the string theory correlator (46) to the $d = 3$ correlator of the type (52) with the generic set of indices $m_j, a_j, b_j (j = 1, 2, 3)$, not all of the ψ -contractions are actually relevant to us. Some of them would result in appearance of the scalar products of the momenta in the 9-derivative contribution. Such terms are of no interest to us since, in the duality context, they would correspond to special degenerate correlators in $d = 3$ where the polarizations of $d = 3$ operators at $\vec{x}_{1,2,3}$ are contracted along one or more mutual directions. On the other hand, we are interested to investigate the relevance of the string correlator (46) to the most general form of the $d = 3$ correlator (52), i.e. in the case with no contractions among m_j, a_j, b_j indices with different j . Furthermore, we assume that all the indices are polarized along the $d = 3$ boundary of AdS_4 . With all these constraints imposed, it is straightforward to check that the only relevant 9-derivative contributions to the correlator (46) stem from the the second part of $G(p)$ -insertion containing $L_a^{(2)}$ -factor, while the first one, with the $L_a^{(1)}$ -factor, only gives rise to degenerate terms with m_j, a_j, b_j -contractions. The reason is that the ψ -correlator pattern for all the terms involving $L_a^{(1)}$, has the form

$$\begin{aligned}
& \lim_{\tau_2 \rightarrow \infty} < \psi^{m_3} \partial^{(p_1)} \psi^{c_3} \partial^{(p_2)} \psi^{d_3}(\tau) \bar{\psi}^m(\bar{z}) \psi^{m_1}(\tau_1) \psi^{m_2}(\tau_2) > \\
& = \tau_2^{-1} (-1)^{p_1+p_2} p_1! p_2! \times \left\{ \frac{\eta^{m_2 m_3} \eta^{d_3 m} \eta^{c_3 m_1}}{(\tau - \bar{z})^{p_2+1} (\tau - \tau_1)^{p_1+1}} - \frac{\eta^{m_2 m_3} \eta^{c_3 m} \eta^{d_3 m_1}}{(\tau - \bar{z})^{p_1+1} (\tau - \tau_1)^{p_2+1}} \right\} + O(\tau_2^{-2})
\end{aligned} \tag{54}$$

leading to unwanted degenerate contractions because of the common $\eta^{m_2 m_3}$ factor.

Straightforward computation of the relevant matter $(X + \psi)$ -part of the integrand of (46)

involving $L_a^{(2)}$ then gives

$$\begin{aligned}
\lim_{\tau_2 \rightarrow \infty} A_{matter}(\tau, z, \bar{z}, \tau_1, \tau_2) &= (3 + 9q^2) e_m^a(p) \omega_{m_3}^{a_3 b_3 | c_3 d_3}(k_1) \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \\
&\times \tau_2^{-4} \times \{ 12 [k_2^{a_3} k_2^{b_3} (\frac{1}{\tau - \tau_1} - \frac{1}{\tau - z} - \frac{1}{\tau - \bar{z}})^2 + k_3^{a_3} k_3^{b_3} (\frac{1}{\tau - z} + \frac{1}{\tau - \bar{z}})^2 \\
&\quad - (k_2^{a_3} k_3^{b_3} + k_3^{a_3} k_2^{b_3}) (\frac{1}{\tau - \tau_1} - \frac{1}{\tau - z} - \frac{1}{\tau - \bar{z}}) (\frac{1}{\tau - z} + \frac{1}{\tau - \bar{z}})] \\
&\quad \times [\frac{2\eta^{mm_2} \eta^{cm_1} \eta^{m_3 d}}{(\tau - \tau_1)^3 (\tau - z)^6} - \frac{\eta^{mm_2} \eta^{dm_1} \eta^{m_3 c}}{(\tau - \tau_1)^4 (\tau - z)^5} \\
&\quad + \frac{4\eta^{mm_3} \eta^{cm_1} \eta^{m_2 d}}{(\tau - \tau_1)^3 (\tau - z)^5 (z - \bar{z})} - \frac{3\eta^{mm_3} \eta^{cm_2} \eta^{m_1 d}}{(\tau - \tau_1)^4 (\tau - z)^4 (\tau - \bar{z})}] \\
&\quad - 12 [k_2^{a_3} k_2^{b_3} (\frac{1}{\tau - \tau_1} - \frac{1}{\tau - z} - \frac{1}{\tau - \bar{z}}) (\frac{1}{(\tau - \tau_1)^2} - \frac{1}{(\tau - z)^2} - \frac{1}{(\tau - \bar{z})^2}) \\
&\quad + k_3^{a_3} k_3^{b_3} (\frac{1}{\tau - z} + \frac{1}{\tau - \bar{z}}) (\frac{1}{(\tau - z)^2} + \frac{1}{(\tau - \bar{z})^2}) \\
&\quad - k_2^{a_3} k_3^{b_3} (\frac{1}{(\tau - \tau_1)} - \frac{1}{(\tau - z)} - \frac{1}{(\tau - \bar{z})}) (\frac{1}{(\tau - z)^2} + \frac{1}{(\tau - \bar{z})^2}) \\
&\quad - k_3^{a_3} k_2^{b_3} (\frac{1}{(\tau - \tau_1)^2} - \frac{1}{(\tau - z)^2} - \frac{1}{(\tau - \bar{z})^2}) (\frac{1}{\tau - z} + \frac{1}{\tau - \bar{z}})] \\
&\quad \times [\frac{2\eta^{mm_2} \eta^{cm_1} \eta^{m_3 d}}{(\tau - \tau_1)^2 (\tau - z)^6} + \frac{4\eta^{mm_3} \eta^{cm_1} \eta^{m_2 d}}{(\tau - \tau_1)^2 (\tau - z)^5 (\tau - \bar{z})} - \frac{2\eta^{mm_3} \eta^{cm_2} \eta^{m_1 d}}{(\tau - \tau_1)^4 (\tau - z)^3 (\tau - \bar{z})}] \\
&\quad + \frac{5}{2} [k_2^{a_3} k_2^{b_3} (\frac{1}{\tau - \tau_1} - \frac{1}{\tau - z} - \frac{1}{\tau - \bar{z}}) (\frac{1}{(\tau - \tau_1)^3} - \frac{1}{(\tau - z)^3} - \frac{1}{(\tau - \bar{z})^3}) \\
&\quad + k_3^{a_3} k_3^{b_3} (\frac{1}{\tau - z} + \frac{1}{\tau - \bar{z}}) (\frac{1}{(\tau - z)^3} + \frac{1}{(\tau - \bar{z})^3}) \\
&\quad - k_2^{a_3} k_3^{b_3} (\frac{1}{(\tau - \tau_1)} - \frac{1}{(\tau - z)} - \frac{1}{(\tau - \bar{z})}) (\frac{1}{(\tau - z)^3} + \frac{1}{(\tau - \bar{z})^3}) \\
&\quad - k_3^{a_3} k_2^{b_3} (\frac{1}{(\tau - \tau_1)^3} - \frac{1}{(\tau - z)^3} - \frac{1}{(\tau - \bar{z})^2}) (\frac{1}{\tau - z} + \frac{1}{\tau - \bar{z}})] \\
&\quad \times [\frac{\eta^{mm_2} \eta^{cm_1} \eta^{m_3 d}}{(\tau - \tau_1)^2 (\tau - z)^5} + \frac{3\eta^{mm_3} \eta^{cm_1} \eta^{m_2 d}}{(\tau - \tau_1)^2 (\tau - z)^4 (\tau - \bar{z})} - \frac{2\eta^{mm_3} \eta^{cm_2} \eta^{m_1 d}}{(\tau - \tau_1)^3 (\tau - z)^3 (\tau - \bar{z})}] \\
&\quad + \frac{57}{8} [k_2^{a_3} k_2^{b_3} (\frac{1}{(\tau - \tau_1)^2} - \frac{1}{(\tau - z)^2} - \frac{1}{(\tau - \bar{z})^2})^2 \\
&\quad + k_3^{a_3} k_3^{b_3} (\frac{1}{(\tau - z)^2} + \frac{1}{(\tau - \bar{z})^2})^2 \\
&\quad + (k_2^{a_3} k_3^{b_3} + k_3^{a_3} k_2^{b_3}) (\frac{1}{(\tau - \tau_1)^2} - \frac{1}{(\tau - z)^2} - \frac{1}{(\tau - \bar{z})^2}) (\frac{1}{(\tau - z)^2} + \frac{1}{(\tau - \bar{z})^2})]
\end{aligned}$$

$$\begin{aligned}
& \times \left[\frac{\eta^{mm_2} \eta^{cm_1} \eta^{m_3d}}{(\tau - \tau_1)^2 (\tau - z)^5} + \frac{3\eta^{mm_3} \eta^{dm_2} \eta^{m_1c}}{(\tau - \tau_1)^2 (\tau - z)^4 (\tau - \bar{z})} - \frac{2\eta^{mm_3} \eta^{cm_2} \eta^{m_1d}}{(\tau - \tau_1)^2 (\tau - z)^4 (\tau - \bar{z})} \right] \} \\
& \times \left\{ [k_1^{a_1} k_1^{b_1} \left(\frac{1}{\tau - \tau_1} + \frac{1}{\tau_1 - z} + \frac{1}{\tau_1 - \bar{z}} \right)^2 + k_3^{a_1} k_3^{b_1} \left(\frac{1}{\tau_1 - z} + \frac{1}{\tau_1 - \bar{z}} \right)^2 \right. \\
& \quad \left. + (k_1^{a_1} k_3^{b_1} + k_3^{a_1} k_1^{b_1}) \left(\frac{1}{\tau - \tau_1} + \frac{1}{\tau_1 - z} + \frac{1}{\tau_1 - \bar{z}} \right) \left(\frac{1}{\tau_1 - z} + \frac{1}{\tau_1 - \bar{z}} \right) \right] \\
& \quad \times [k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_2^{b_2} + k_1^{a_2} k_2^{b_2} + k_1^{b_2} k_1^{a_2}] \times \frac{(k_1^a + k_2^a + k_3^a)}{z - \bar{z}} \} \\
& \times |\tau - z|^{-2k_1 k_2 - 2k_1 k_3} (\tau - \tau_1)^{k_1 k_2} |\tau_1 - z|^{-2k_1 k_2 - 2k_2 k_3} + O(\tau_2^{-5})
\end{aligned} \tag{55}$$

where we used the momentum conservation along with the on-shell conditions on the space-time fields. The next step is to perform the integrations in τ and z . We start with the integral over τ which, upon the conformal transformation (47), is along the real line. As was mentioned above, a convenient choice for τ_1 is $\tau_1 = -i$ on the disc corresponding to $\tau_1 = 0$ on the half-plane. The overall integral is given by

$$A(p; k_1, k_2, k_3) = \frac{1}{2!6!} \int_{-\infty}^{\infty} d\tau \tau^6 \int d^2 z z^2 \{ A_{matter}(\tau, z, \bar{z}) A_{gh}(\tau, z) \} \tag{56}$$

where

$$A_{matter}(\tau, z, \bar{z}) A_{gh}(\tau, z) \equiv A_{matter}(\tau, z, \bar{z}, \tau_1, \tau_2) A_{gh}(\tau, z, \tau_1, \tau_2) |_{\tau_1=0, \tau_2 \rightarrow \infty} \tag{57}$$

and we used the fact that the leading order $\sim \tau_2^4$ -factor of the ghost part of the correlator is cancelled by the leading order $\sim \tau_2^{-4}$ -factor of its matter part. The τ integral in (46) looks tricky to evaluate. However, for our purposes we only need its asymptotic value in the field theory limit, that is, in the leading order of α' . In this limit the τ integral is dominated by contributions from the region $\tau \sim z$ as the integrand becomes singular when z approaches the real axis. In this case, we shall use the asymptotic formula

$$\begin{aligned}
& \lim_{\epsilon \rightarrow 0} \int d\tau \int d^2 z f(z, \bar{z}) g(\tau, \bar{z}) (\tau - z)^{\epsilon - N} \\
& \sim \frac{(-1)^{N-1}}{(N-1)! \epsilon} \left\{ \int d^2 z f(z, \bar{z}) \partial^{(N-1)} g(z, \bar{z}) + O(\epsilon) \right\}
\end{aligned}$$

(where $\epsilon \equiv -k_1 k_2 - k_1 k_3 = k_2 k_3$) The result (which is valid up to subleading α' -corrections) is given by the lengthy expression (function in z and \bar{z}) presented in the

Appendix. Finally, it remains to evaluate the z -integral of (62), (63). The integrand of (63) is cumbersome but structurally all of the terms are of the type:

$$I(k_1, k_2, k_3) \sim (k_2 k_3)^{-1} \int d^2 z \{ z^{-k_2 k_3 + N_1} \bar{z}^{-k_1 k_2 - k_1 k_3 + N_2} (z - \bar{z})^{-k_1 k_2 - k_1 k_3 - N_3} + c.c. \} \quad (58)$$

where $N_{1,2,3}$ are some integer numbers, different for each of the terms entering (63). The integrals of the type (58) are over the upper half-plane and are still tedious to evaluate. It is therefore convenient, by using the overall conformal invariance of the overall correlator (46) to conformally map it back to the unit disc $(z, \bar{z}) \rightarrow (u, \bar{u})$ and introducing $u = r e^{i\alpha}$ for the disc coordinates. Then, the transformation (47) reduces the integrals of the type (58) to those of the generalized elliptic type:

$$I(k_1, k_2, k_3) \sim 2^{-N_1 - N_2} \int_0^1 dr r \left(\frac{r^2 - 1}{r^2 + 1} \right)^{N_1 + N_2 - N_3 - 2k_1 k_2 - 2k_1 k_3 - 2k_2 k_3} \times \int_0^{2\pi} d\alpha \left(1 + \frac{2r}{r^2 - 1} \cos \alpha \right)^{-k_1 k_3 - k_2 k_3 + N_1} \left(1 - \frac{2r}{r^2 - 1} \cos \alpha \right)^{-k_1 k_2 - k_2 k_3 + N_2} \quad (59)$$

$$+ (N_1 \leftrightarrow N_2, k_2 \leftrightarrow k_3)$$

The overall amplitude (46) is then given by the lengthy expression (63) described in the Appendix; the answer, however, simplifies in the field theory limit of $\alpha' \rightarrow 0$. Integrating the amplitude (46) over k_1, k_2, k_3 and p , using the momentum conservation in the 5-point amplitude that eliminates the integral over p , the asymptotics of (46) in the field theory limit gives:

$$A(k_1, k_2, k_3) \sim (k_1 k_2)^{-1} (k_1 k_3)^{-1} (k_2 k_3)^{-1} \omega_{m_1}^{a_1 b_1}(k_1) \omega_{m_2}^{a_2 b_2}(k_2) \omega_{m_3}^{a_3 b_3}(k_3) \times \{ k_2^{m_1}(k_2)_{a_1}(k_2)_{b_1}(k_3)^{m_2}(k_3)_{a_2}(k_3)_{b_2}(k_1)^{m_3}(k_1)_{a_3}(k_3)_{b_2} + k_3^{m_1}(k_3)_{a_1}(k_3)_{b_1}(k_3)^{m_2}(k_3)_{a_2}(k_3)_{b_2}(k_2)^{m_3}(k_2)_{a_3}(k_2)_{b_2} \} + \dots \quad (60)$$

where we skipped the contact terms (proportional to the delta-functions in the position space), used the zero torsion conditions (37), (40) relating $\omega^{2|2}$ to $\omega^{2|0}$ along with the on-shell conditions for ω 's and neglected the contributions in the subleading order in α' .

This, up to an overall normalization and the contact terms (proportional to the delta-functions in the position space), coincides with the correlator (52) transformed to the momentum space.

Next, evaluating the lower derivative terms in the amplitude (56) gives the answer proportional to

$$A_{lower-der.}(k_1, k_2, k_3) \sim (15 - 6q^2) I_{lower-der}(k_1, k_2, k_3) + \dots \quad (61)$$

with the contact terms skipped. The explicit expression for $I_{lower-der}(k_1, k_2, k_3)$ is given in the Appendix; (61) particularly implies that, apart from the contact terms, the only lower derivative contribution to the overall amplitude (46) is proportional to the factor of $\sim 15 - 6q^2$ which stems from q -independent and q -dependent $L_a^{(1)}$ and $L_a^{(2)}$ pieces of the closed string insertion for the vielbein vertex operator (satisfying the AdS vacuum solution in the leading order of the beta-function). This expression vanishes (up to contact terms and those of the higher order in α') in $d = 4$ where $q = \sqrt{\frac{9-d}{2}} = \sqrt{\frac{5}{2}}$. This means that in the special case of $d = 4$ only the nine-derivative contribution survives in the string-theoretic amplitude (46) which gives precise holographic relation between AdS string sigma-model (41) in the case of $d = 4$ and the dual free field theory correlator (53) in $d = 3$ for spin 3. Thus the AdS_4/CFT_3 holography correspondence for higher spins appears to be surprisingly related to the value of the Liouville background charge value in $d = 4$ which stems from two-dimensional CFT. This fact is by itself quite intriguing and definitely needs further investigation.

5. Conclusion and Discussion

In this paper we analyzed the AdS_4/CFT_3 higher spin holography using string-theoretic sigma-model describing gravity and higher spin perturbations around AdS background in the low-energy limit. We found that, in the leading order in α' and cosmological constant, the three-point correlator for spin 3 particles in AdS_4 reproduces the free-field correlator for the large N vector model in $d = 3$. Surprisingly, we found that this holographic correspondence appears to be related to the value of the Liouville background charge in $d = 4$ which allows for cancellation of the lower-derivative terms (up to contact terms), so the leading order of AdS string theory appears to be in agreement with Maldacena-Zhiboedov's proposal [16], [17]. One definitely needs to check whether such a cancellation also holds for vertex operators for spins greater than 4 and for higher point correlators in the AdS_4 case. On the other hand, the lower derivative terms do persist in the three-point amplitudes for $d \neq 4$. This is the signal that the higher spin / CFT holography has more complicated character in higher dimensions, where the dual theories are no longer free. Moreover, even in AdS_4 the string theory corrections may definitely modify the limit in which Maldacena-Zhiboedov's theorem holds. We hope to implement these computations in the future papers. The results of this paper suggest that string theoretic approach may provide interesting insights to HS/CFT duality, such as the relevance of the Liouville theory to the $d = 4$ case. It would be interesting to see possible relations

of this fact to the *AGT* conjecture since open string amplitudes for spin 1 in the sigma-model (41) should particularly involve the super Yang-Mills theory in the low energy limit. Another question of immediate interest is to use the sigma-model (41) in order to study the AdS_5/CFT_4 holography for higher spins. To approach this problem, one has to study the lower derivative terms appearing in the sigma-model correlators, as well as the higher order corrections in the cosmological constant. Finally, in the AdS_4/CFT_4 case it would be of crucial importance to understand the relation between string-theory formalism and the twistor space approach used by Vasiliev [18] to study the higher spin holography. This relation may probably, in some form or another, involve the modifications of twistor string theory developed by Witten [54] This altogether gives the list of problems to address in the future which of course is still very preliminary and incomplete.

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Appendix

In this section we present explicit expressions for the amplitude (46) leading to the asymptotics (60). To abbreviate the expressions, we adopt the following notations:

$$\begin{aligned}
a &\equiv z \\
b &\equiv \bar{z} \\
c &\equiv k_1^{a_1} k_1^{b_1} \\
d &\equiv k_1^{a_1} k_3^{b_1} + k_1^{b_1} k_3^{a_1} \\
f &\equiv k_2^{a_3} k_2^{b_3} \\
g &\equiv k_2^{a_3} k_3^{b_3} + k_3^{a_3} k_2^{b_3} \\
h &\equiv k_3^{a_3} k_3^{b_3} \\
k &\equiv k_3^{a_1} k_3^{b_1} \\
p &\equiv \eta^{mm_2} \eta^{cm_1} \eta^{m_3 d} \\
q &\equiv \eta^{mm_3} \eta^{cm_1} \eta^{m_2 d} \\
t &\equiv \eta^{mm_3} \eta^{cm_2} \eta^{m_1 d} \\
u &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \\
&\quad \times (k_1^a + k_2^a + k_3^a) \\
&\quad \times (k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_1^{b_2} k_1^{a_2}) \\
L_1 &= k_1 k_2 + k_1 k_3 \\
L_2 &= k_1 k_2 + k_2 k_3 \\
L_3 &= k_1 k_3 + k_2 k_3
\end{aligned} \tag{62}$$

Then the amplitude can be expressed covariantly in a convenient way, suppressing the indices, in terms of the variables $a, b, c, d, f, g, h, k, q, t, u, L_{1,2,3}$. The evaluation of the τ

integral then leads to the following answer:

$$\begin{aligned}
A(k_1, k_2, k_3) = & u \int da db \left\{ \frac{f a^{L_3} b^{L_2}}{16(a-b)^{19+L_1}} \right. \\
& \times [ab(-8a^{13}(b(3592376k + 203232c - 470026d) - 45(1488k + 122c - 231d))t + \\
& 24a^{14}(1488k + 122c - 231d)t + 10b^{13}(4k + c - d)(-576(-20 + 13b)p \\
& + 38925q - 29952bq - 66270t + 32448bt) + a^9b^4(2(b(-27583068k + 2090940c - 780802d) \\
& - 15(-71416k + 6175c + 249d)) - 384((-53420k + 736c - 491d))p - 2414880kq + \\
& 89547132bq + 595980cq - 13527948bcq - 188055dq + 8919888bdq - 38145120kt \\
& + 442112208bkt - 6170220ct + 40507190bct + 4309920dt - 111451878bdt) + \\
& a^6b^7((b(36984432k + 5216119c - 22448132d) - 30(952240k + 55785c - 179903d)) - \\
& 384(b(-31790k + 873c - 13993d) + 105(-16k + 5c + 26d))p + \\
& 79359120kq - 51353040bkq + 4399245cq - 7162686bcq \\
& - 12648735dq + 34253916bdq - 234435600kt + 204418288bkt \\
& + 8927160ct - 48353746bct + 26078880dt - 72924756bdt) \\
& + a^{10}b^3(2(b(37160040k + 1092363c - 3283276d) - \\
& 15(114032k + 3351c - 10653d)) - 192(b(36512k + 782c - 2099d) - 30(80k + c - 4d))p + \\
& 10582200kq - 233099616bkq + 302985cq - 6588324bcq - 976095dq + 20179833bdq \\
& - 102815400kt + 1209791888bkt - 1626570ct + 30153268bct + 7224210dt - 59663780bdt) \\
& + a^3b^{10}((2b(21928972k + 8677069c - 7234681d) - \\
& 15(1299632k + 41244c - 216481d)) - 192(-22800k + 19668bk - 3540c \\
& + 34411bc + 6495d - 19339bd)p + 31623120kq - 63763176bq + 2049300cq - \\
& 28605954bcq - 5653440dq + 22235154bdq - 29434440kt + \\
& 37177976bkt - 1858710ct + 40276674bct + 6241860dt - 23223764bdt) \\
& - 10a^2b^{11}((b(2054748k + 414913c - 470605d) + \\
& 12(-4456k + 17251c - 7576d)) - 96(-2688k + 8332b + 348c \\
& + 2525bc + 252d - 2369bd)p + 203220kq - 3367224bkq \\
& - 243405cq - 865908bcq + 74475dq + 871407bdq - 695820kt \\
& + 3149708bkt + 203355ct + 1343541bct + 57525dt - 1109229bdt) \\
& \left. \right\}
\end{aligned}$$

$$\begin{aligned}
& -10ab^{12}((-30(12076k + 3989c - 3504d) + \\
& 13b(1044k + 2573c - 1417d)) + 192(13b(60k + 7c - 11d) \\
& + 75(4k + 3c - 2d))p + 355680kq + 391248bkq + 169920cq \\
& + 30342bcq - 129420dq - 64077bdq + 135120kt - 1255384bkt - 168120ct - \\
& 198978bct + 67170dt + 256412bdt) \\
& + a^{11}b^2((b(9576896k + 246342c - 898206d) - \\
& 15(15472k + 542c - 1683d)) + 192(2b(c + 194(-20k + d)) - 15(-16k + d))p \\
& + 696240kq - 28709400bkq + 24390cq - 738933bcq - 75735dq \\
& + 2693463bdq + 18384600kt - 181700576bkt + 205590ct \\
& - 11289448bct - 556680dt + 6229332bdt) + a^4b^9((30(438608k + 139259c - 170889d) \\
& + b(31195608k - 23625295c + 7077692d))192(30(496k - 245c + 130d) \\
& + b(-109400k + 42721c + 1074d))p - 34454160kq \\
& - 50747856bkq - 8345745cq + 36016704bcq + 10265985dq - 9433944bdq \\
& + 73283520kt + 39522728bkt + 15467220ct - 75827612bct - 19635420dt + 22658110bdt) \\
& + a^5b^8((-30(-993296k + 65495c + 51431d) + b(-140574840k + 8076197c + 20592266d)) \\
& - 192(-420(-100k + 11c + 11d) + b(-154056k + 21149c + 35272d))p - 53598240kq \\
& + 278165748bkq + 3934800cq - 4726128bcq + 2016675dq - 42460149bdq \\
& + 63153600t - 439526024bt - 21984180ct + 63570616bct + 8993340dt + 52194190bdt) \\
& + a^8b^5((30(632960k + 25861c - 55751d) + b(-155995128k - 11490326c + 21567689d)) \\
& + 192(b(42216k + 6692c - 19703d) - 30(320k + 23c - 34d))p - 55011960q \\
& + 441897552bq - 2069865cq + 30049308bcq + 4492845dq - 53982408bdq + 247050960kt \\
& - 1075703328bkt + 12480240ct - 16542330bct - 12267000dt + 64259624bdt) \\
& + a^7b^6((30(-527920k + 21473c - 22362d) + b(109033376 - 1998049c + 328756d)) \\
& - 192(b(225668k + 1180c - 13133d) - 30(928k + 22c + 13d))p \\
& + 27250560kq - 132110208bkq - 2015190cq + 4628397bcq + 2606355dq - 9431328bdq \\
& - 13143120kt + 158121584bkt - 5041440ct + 2368354bct - 18233580dt + 75056032bdt) \\
& + a^{12}b(120(105076k + 4767c - 12401d)t + b(-192(-16 + d)p + 46416kq
\end{aligned}$$

$$\begin{aligned}
& +1626cq - 5049dq - 347866128kt - 12118888ct + 34908536dt))) \\
& + \left\{ \frac{ha^{L_3}b^{L_2}}{8(a-b)^{19+L_1}} \times \right. \\
& [ab(4a^{14}(8462k + 577c - 1202d)t + 2a^{13}(30(8462k + 577c - 1202d) \\
& + b(-12742604k - 632218c + 1559205d))t + 30b^{13}(4k + c - d)((960 - 416b)kp \\
& + 7005kq - 4108bkq - 7260kt + 2106bkt) + a^9b^4(2(4b(-1601611k + 615629c - 659330d) \\
& + 15(4848k - 7679c + 6613d)) - 192(-90c + b(-77608k + 1129c - 646d) \\
& + 60(81k + d))p - 348480kq + 36705972bkq + 686340cq - 14675466bcq \\
& - 583785dq + 15626610bdq - 23306640kt + 389991016bkt - 3838620ct + 22950092bct \\
& + 3457260dt - 73172098bdt) + \\
& a^{10}b^3((b(79470528k + 2016958c - 6521170d) - 30(123808 + 3051c - 10481d)) \\
& - 96(b(40888k + 780c - 2883d) - 30(100k + c - 5d))p + \\
& 11146320kq - 238310688bkq + 274005cq - 6035748bcq - 942075dq + 19516749bdq \\
& - 77125680kt + 841941584bkt - 1654440ct + 26608962bct + 5691120dt - 50462452bdt) \\
& + a^6b^7((b(23733672k + 3102799c - 8854348d) - 30(889920k + 38513c - 122707d)) \\
& - 384(b(-46491k + 10514c - 1893d) + 105(28k - 5c + 13d))p + 76062960kq \\
& - 42926256bkq + 3536955cq - 9361710bcq - 10353825dq + 17679426bdq - 139153920kt \\
& + 36775152bkt + 875940ct - 7023550bct + 12947220dt - 5302270bdt) \\
& + 30ab^{12}((13b(3084k + 355c - 563d) + 160(44k + 57c - 34d)) \\
& - 32(45(-4k + c) + 13b(60k + 11c - 13d))p + 44880kq \\
& - 154960bkq - 7500cq - 26598bcq - 1860dq + 32669bdq \\
& - 193440kt + 246376bkt - 19440ct + 49010bct + 33900dt - 55302bdt) \\
& + a^4b^9((30(443488k + 70935c - 97085d) + b(-1151176k - 7947881c + 3618600d)) \\
& - 96(b(143928k + 7821c - 28474d) + 60(-512k + 133c - 2d))p - 35452800q \\
& + 1654464bq - 5650695cq + 11720508bcq + 7261695dq - 4865790bdq \\
& + 44217240kt - 31887760bkt + 4031520ct + 100832bct - 5912910dt + 247636bdt) \\
& + a^{11}b^2((b(9997696k + 207342c - 832814d) - 1155(224k + 6c - 21d)) \\
& + 96(-15(-24k + d) + 2b(-5444k + c + 193d))p + 776160kq
\end{aligned}$$

$$\begin{aligned}
& -29992848bkq + 20790cq - 622065bcq - 72765dq + 2498523bdq \\
& +11616720t - 103495104bt + 249960ct - 9790670bct - 410820dt + 6061460bdt) \\
& +a^5b^8((30(314384k - 51493c + 13147d) + b(-69230024k - 155149c + 9845590d)) \\
& +96(210(7c + 40(-5k + d)) + b(85184k + 39575c - 45454d))p - 24069600kq \\
& +172712364bkq + 3804840cq + 9101964bcq - 925515dq - 26732397bdq \\
& -1771320kt - 129295912bkt - 7515360ct - 9836246bct + 6549780dt + 19131876bdt) \\
& +a^8b^5((30(606368k + 18269c - 47855d) + b(-149759256k - 7429006c + 16898695d)) \\
& +96(b(-21696k + 7168c - 32107d) - 300(20k + 2c - 7d))p \\
& -53472600kq + 431464896bkq - 1644975cq + 22336260bcq + 4232835dq - 49353342bdq \\
& +162720240kt - 573713128bkt + 7605180ct + 2790792bct - 10584600dt + 27614802bdt) \\
& +a^{12}b(30(357076k + 15242c - 40239d)t + b(-96(-24k + d)p + 51744kq \\
& +1386cq - 4851dq - 283508960kt - 9804142ct + 27924468dt)) \\
& -2a^2b^{11}(-3(16(b(21044k + 8251c - 6951d) - 30(572k + 83c - 123d))p \\
& +(13b(96672k + 48818c - 38053d) + 480(304k + 51c + 4d))q \\
& +2(5(-8316k + 27871c - 19191d) + b(842212k - 186857c + 3817d))t)) \\
& +a^3b^{10}(-2(48(b(-35828k + 11269c - 571d) - 75(184k + 106c - 89d))p \\
& +3(b(3524768k + 1583210c - 1123521d) - 60(29492k + 3877c - 5475d))q \\
& +2(b(7128476k + 410553c - 1514936d) + 45(-57308k + 3915c + 6786d))t)) \\
& +a^7b^6(3(32(-30(-1192k + 17c + 53d) + b(-257540k + 10748c + 41515d))p \\
& +(b(1784512k + 4673403c - 6328878d) + 15(259520k - 56778c + 80197d))q \\
& -2(b(18432028k + 2063821c - 6743278d) + 10(-140688k + 6877c + 155327d))t))))] \\
& +[\frac{ga^{L_3-3}b^{L_2}}{8(a-b)^{19+L_1}}((96a^{20} + 2a^3(4835 - 8208b)b^{16} \\
& +2a(95 - 48b)b^{18} - 10b^{19} + 114a^2b^{17}(-15 + 16b) - 2a^{19}(5 + 912b) \\
& +2a^{18}b)(3248k - 371c + 575d)t + a^{16}b^2(-30(102632k + 6640c - 14211d)t \\
& +b(96(-20k + d)p - 37500kq - 1116cq + 3735dq + 101842384kt \\
& +4629100ct - 11840166dt))) \\
& +a^{10}b^8(b(96(-124772k + 21901c - 17779d)p
\end{aligned}$$

$$\begin{aligned}
& +69337080kq - 25027488cq + 22411995dq + 47573000kt + 30096024ct \\
& -71921924dt) - 15(672(-40k + 5c - 52d)p + 4659444kq + 5988cq \\
& -137073dq - 2235256kt - 50576ct - 607218dt)) \\
& +2a^4b^{14}(150(4k + c - d)(-9(16p + 79q - 64t)) \\
& -5b(-2496(4k + c - d)p - 26988kq - 6747cq + 6747dq + 7280kt + 1820ct - 1820dt)) \\
& +a^{12}b^6(-b(1440(684k + 462c - 1727d)p + 385641132kq + 4095636cq \\
& -12983877dq + 94219368kt + 9469882ct - 66033204dt) \\
& -15(-96(520k + 43c - 104d)p - 34077cq + 136692dq + 1857640kt \\
& -22998ct + 47306dt)) \\
& +a^{15}b^3(-96(-9324k + 2c + 387d)p + 22208160kq + 500691cq \\
& -1942140dq + 239709600kt + 5842382ct - 16780882dt) + 15(96(-20k + d)p \\
& -1116cq + 3735dq - 809096kt - 14066ct + 63272dt) \\
& +a^8b^{10}(96(-1520k + 511c - 134d)p - 229047cq \\
& -63054dq + 661742ct - 567764dt) + b(-192(-63332k + 8725c + 7387d)p \\
& +6143760cq + 14083365dq + 160389568kt - 33349956ct + 1211602dt) \\
& +a^{14}b^4(-96(180k + 2c - 9d)p - 620448kq - 14901cq + 52512dq + 910496kt \\
& +17798ct - 131728dt) + b(96(38700k + 781c - 2491d)p + 213241020kq + 4928781cq \\
& -16835793dq - 178959408kt + 2472926ct + 11129654dt) \\
& -a^7b^{11}(15(96(1220k + 383c - 439d)p - 2448324kq \\
& -261879cq + 501360dq + 4165984kt + 202016ct - 700056dt) \\
& +b(480(-1616k + 4568c - 1991d)p + 38861664kq + 13797912cq \\
& -13942029dq - 86969480kt - 23944502ct + 25663916dt)) \\
& +a^9b^9(-672(-400k + 29c + 62d)p - 3995256kq - 199923cq \\
& +725346dq + 8950184kt - 237858ct - 907442dt) \\
& +b(-96(119620k + 9213c - 40363d)p + 49804440kq + 2624862cq \\
& -20884221dq - 231609256kt + 22319212ct + 37755238dt) \\
& -a^{11}b^7(480(424k + c - 8d)p - 2963400kq - 187257cq
\end{aligned}$$

$$\begin{aligned}
& +419913dq + 9062904kt + 270722ct - 497360dt) \\
& +b(384(-60401k + 1196c + 6831d)p \\
& +461626416kq + 17580171cq - 64284969dq \\
& -722013808kt + 14928440ct + 42684832dt) \\
& -2a^{17}b^2((-3097048k - 244028c + 464157d)t \\
& -2a^5b^{13}(96(4k + 9c - 5d)p - 58884kq - 8973cq \\
& +11847dq + 70960kt + 11540ct - 14640dt) + 65b(-2(144(20k + 3c - 4d)p \\
& +9(3604k + 743c - 822d)q - 32588kt - 6995ct + 7571dt))) \\
& +a^{13}b^5((-96(-520k + 11c - 4d)p - 790128kq - 44247cq \\
& +73722dq + 4384696kt + 124148ct - 207996dt) + \\
& b(2(55580k + 48(1865c - 3(43676k + 379d))p \\
& +6(16303472k + 1230680c - 2118925d)q \\
& -407402424kt - 14101721ct + 23167123dt))) \\
& +a^6b^{12}(-15(-480(204k + 5c - 33d)p + 633804kq - 115959cq + 4059dq \\
& -338248kt + 265498ct - 122678dt) \\
& +b(-2(96(15676k + 5219c - 4699d)p - 3(5168668k + 279987c - 754942d)q \\
& +20((913672k + 50071c - 134207d)t)))))] + \{a \leftrightarrow b\}
\end{aligned} \tag{63}$$

Next, we perform the conformal transformation from the half-plane coordinates $z \equiv a, \bar{z} \equiv b$ to the r, α coordinates in the disc ($0 \leq r \leq 1, 0 \leq \alpha \leq 2\pi$) using the prescription (58)-(59) and replacing each of the half-plane integrals of the type $\int dadb(a-b)^{\gamma_1}a^{\gamma_2}b^{\gamma_3}$ with corresponding integrals on the disc. Straightforward evaluation of the asymptotics of the disc integrals in the field theory limit $\alpha' \rightarrow 0$ then gives leads to (60), skipping the contact terms. Finally, the lower-derivative contributions to the correlator (46) are given by the overall expression (61) that vanishes in four dimensions. For completeness, we finish with presenting explicit expressions for $I_{lower-der}(k_1, k_2, k_3)$ entering (61). It is convenient to cast it according to

$$I_{lower-der}(k_1, k_2, k_3) = I_3 + I_5 + I_7 \tag{64}$$

with I_p being the p -derivative pieces. The integration procedure is then identical to the one explained for the 9-derivative contribution. Introducing further convenient abbreviations:

$$\begin{aligned}
u_1 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_2}(k_1^a + k_2^a + k_3^a)(k_1^{b_2} + k_2^{b_2} + k_3^{b_2}) k_1^{b_1} \\
u_2 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_2}(k_1^a + k_2^a + k_3^a)(k_1^{b_2} + k_2^{b_2} + k_3^{b_2}) k_3^{b_1} \\
v_1 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a} \\
&\times (k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_2^{b_2} k_2^{a_2}) k_1^{b_1} \\
v_2 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a} \\
&\times (k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_2^{b_2} k_2^{a_2}) k_2^{b_1} \\
v_3 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a} \\
&\times (k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_2^{b_2} k_2^{a_2}) k_3^{b_1} \\
\lambda_1 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_3} \\
&\times (k_1^a + k_2^a + k_3^a)(k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_2^{b_2} k_2^{a_2}) k_1^{b_1} k_2^{b_3} \\
\lambda_2 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_3} \\
&\times (k_1^a + k_2^a + k_3^a)(k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_2^{b_2} k_2^{a_2}) k_3^{b_1} k_2^{b_3} \\
\lambda_3 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_3} \\
&\times (k_1^a + k_2^a + k_3^a)(k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_2^{b_2} k_2^{a_2}) k_1^{b_1} k_3^{b_3} \\
\lambda_4 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_3} \\
&\times (k_1^a + k_2^a + k_3^a)(k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_2^{b_2} k_2^{a_2}) k_3^{b_1} k_3^{b_3} \\
\rho_1 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a a_3} \\
&\times (k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_2^{b_2} k_2^{a_2}) k_2^{b_3} \\
\rho_2 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a a_3} \\
&\times (k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_2^{b_2} k_2^{a_2}) k_3^{b_3}
\end{aligned} \tag{65}$$

the asymptotics of the 7-derivative contribution is computed to give

$$\begin{aligned}
I_7 \sim & (k_1 k_2)^{-1} (k_1 k_3)^{-1} (k_2 k_3)^{-1} \\
& \times \{ u_1 (1493472p + 1195776q + 505040t) + g(2051748p + 2154326q + 908764t) \\
& + h(2214564p + 4140804q + 764500t) - u_2 (f(3649254p + 1628060q + 492190t) \\
& + g(3166474p + 1040800q + 30605620t) + h(2082610p - 2427639q + 1648244t)) \\
& + v_1 (f(132128p + 2003462q + 808692t) + g(988622p + 8267294q + 3016884t) \\
& + h(-4428732p + 10608920q - 764824t)) + v_2 (f(8467662p + 5883454q + 936720t) \\
& + g(12845628p + 9832562q + 964355t) + h(20605796p + 1557672q - 2062380t)) \\
& + v_3 (f(7884240p - 3167390q - 1010042t) + g(8690112p + 40306q + 2087600t) \\
& + h(-3860368p - 894766q + 7057t)) - \lambda_1 (9085p - 30492495q + 735944t) \\
& + \lambda_2 (2235256p - 41879080q - 538906t) + \lambda_3 (1264326p + 24246476q + 426t) + \\
& \lambda_4 (130p + 284472q + 3280845t) + \rho_1 (c(20047925p + 2823510q + 724612t) \\
& + d(2240244p + 3171313q + 462884t) + k(1021245p + 2436124q + 6523480t)) \\
& + \rho_2 (c(-4086001p + 2049050q + 206800t) + d(2006114p - 929683q + 96492t) \\
& + k(-24p + 907200q + 1920160t)) \} + \dots
\end{aligned} \tag{66}$$

Next, to describe the 5-derivative piece, we shall adopt the notations:

$$\begin{aligned}
\gamma_1 & \equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_2} \eta^{a a_3} (k_1^{b_2} + k_2^{b_2}) k_1^{b_1} k_2^{b_3} \\
\gamma_2 & \equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_2} \eta^{a a_3} (k_1^{b_2} + k_2^{b_2}) k_3^{b_1} k_2^{b_3} \\
\gamma_3 & \equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_2} \eta^{a a_3} (k_1^{b_2} + k_2^{b_2}) k_1^{b_1} k_3^{b_3} \\
\gamma_4 & \equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_2} \eta^{a a_3} (k_1^{b_2} + k_2^{b_2}) k_2^{b_1} k_3^{b_3} \\
\delta_1 & \equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{b_1 b_3} \eta^{a a_1} \\
& \times (k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_1^{b_2} k_1^{a_2}) k_2^{a_3} \\
\delta_2 & \equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{b_1 b_3} \eta^{a a_1} \\
& \times (k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_1^{b_2} k_1^{a_2}) k_3^{a_3} \\
\epsilon_1 & \equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{b_1 b_3} \eta^{a_1 a_3} \\
& \times (k_1^{a_2} k_1^{b_2} + k_2^{a_2} k_1^{b_2} + k_1^{a_2} k_2^{b_2} + k_1^{b_2} k_1^{a_2}) (k_1^a + k_2^a + k_3^a)
\end{aligned} \tag{67}$$

Then the asymptotics of the 5-derivative contribution is computed to give

$$\begin{aligned}
I_5 &\sim (k_1 k_2)^{-1} (k_1 k_3)^{-1} (k_2 k_3)^{-1} \\
&\times \{ \gamma_1 (1864828p + 2866523q - 303t) + \gamma_2 (20078254p + 652q - 1887348t) \\
&+ \gamma_3 (1542620p - 1288760q - 1052t) + \gamma_4 (30884240p + 6268906q + 9208t) \\
&+ \delta_1 (2305266p + 764080q - 2006875t) + \delta_2 (2640884p + 306708q + 9861808t) \\
&+ \epsilon_1 (3077454p - 708616q - 979072t) \} + \dots
\end{aligned} \tag{68}$$

Finally, to describe the 3-derivative contributions (up to contact terms) we denote

$$\begin{aligned}
\sigma_1 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_2} \eta^{a a_3} \eta^{b_1 b_2} k_2 \\
\sigma_2 &\equiv \omega_{m_1}^{a_1 b_1}(k_2) \omega_{m_2}^{a_2 b_2}(k_3) \omega_{m_3}^{a_3 b_3 | cd}(k_1) \eta^{a_1 a_2} \eta^{a a_3} \eta^{b_1 b_2} k_3
\end{aligned} \tag{69}$$

Then the asymptotics of the 3-derivative contribution is computed to give

$$\begin{aligned}
I_3 &\sim (k_1 k_2)^{-1} (k_1 k_3)^{-1} (k_2 k_3)^{-1} \\
&\times \{ \sigma_1 (-10887660p + 30458284q + 41010562t) \\
&+ \sigma_2 (10865492p - 30460944q - 41010934t) \} + \dots
\end{aligned} \tag{70}$$

This concludes the evaluation of $I(k_1, k_2, k_3)$ factor describing the lower derivative contributions to the correlator (46), modulo contact terms.

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